

Oberfläche in erster Näherung alle Spannungskomponenten mit Ausnahme von σ_{zz} verschwinden, so daß in dieser Näherung auf der rechten Seite von (19) nur das eine Glied rechts unten übrig bleibt. Dieses liefert, wie bei der Platte, die Dehnungsschwingungen mit einem Dispersionsgesetz

$$\omega = c_{st} \kappa \quad \text{mit} \quad c_{st} = c_t \sqrt{\frac{3c_l^2 - 4c_t^2}{c_l^2 - c_t^2}} = \sqrt{\frac{E}{\rho}}, \quad (20)$$

während man die Biegungsschwingungen auch hier erst in nächster Näherung erhält, mit einem Dispersionsgesetz $\omega \sim \kappa^2$.

Was schließlich noch die nächste Näherung der Schwingungsgleichungen für dünne Platten betrifft, so hat man nur die nächste Näherung der Bedingungsgleichungen (8) und (9) für die beiden Hauptäste zu ermitteln und dann die in der Dispersionsformel stehenden Größen ω bzw. κ durch die Operatoren $i\partial/\partial t$ bzw. $-i\partial/\partial x$ zu er-

setzen. Da aus (8) in nächster Näherung

$$\omega^2 = \kappa^2 c_{Pl}^2 \left(1 - \frac{\kappa^2 h^2}{3} \left(\frac{c_l^2 - 2c_t^2}{c_l^2} \right)^2 + \dots \right) \quad (21)$$

an Stelle von (13) mit (12) folgt, und da ebenso aus (9)

$$\omega^2 = \frac{\kappa^4 h^2}{3} c_{Pl}^2 \left(1 - \frac{\kappa^2 h^2}{15} \frac{27c_l^2 - 20c_t^2}{c_l^2} + \dots \right) \quad (22)$$

an Stelle von (14) wirksam wird, erhält man in dieser nächsten Näherung für die Dehnungsschwingungen eine Differentialgleichung, die in den räumlichen Ableitungen von vierter Ordnung ist, und für die Biegungsschwingungen eine Gleichung mit den sechsten räumlichen Ableitungen. Dabei bleiben die Zeitableitungen nach wie vor von zweiter Ordnung.

The earth's constants from combined electric and magnetic measurements partly in the vicinity of the emitter

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In this article a way is indicated for the determination of the soils constants, based upon measurements of both the electric and the magnetic field (remote from but also in the vicinity of the emitter). The method is based especially upon the different behaviour of the electric and magnetic field as a function of the distance in the vicinity of the emitter. Owing to this difference and its mathematical form it will be possible to derive the required constants of the earth by means of *one simple system of curves, which may be used again in every other case, as the curves do not depend upon the distances of the points of observation from the emitter*. This greatly simplifies the solution of the problem.

§ 1. The field on the earth at a distance from the emitter of the order of a few wavelengths

The dielectric constant ϵ and the conductance σ of the earth have sometimes been derived from measurements of the fieldstrength at great distances from the emitter. In that case mostly long waves were considered where the conduction currents in the earth prevailed over the displacement currents.

We shall now investigate the possibility of arriving at the same result from measurements of the fieldstrength, partly made in the vicinity of,

¹ In grateful remembrance of a year spent in Munich as a Rockefeller-fellow.

partly at greater distances from the emitter, and even without assuming the conduction current to be great with respect to the displacement current.

With "vicinity of the emitter" we here mean distances r , measured along the surface of the earth and of the order of two to four wavelengths:

$$2\lambda < r < 4\lambda.$$

We assume that even at these short distances the emitter may be considered as a vertical mathematical dipole, situated close to the ground.

This will not be the case for the real emitter, to be built in a special place for transmitting or broadcasting purposes.



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Therefore we imagine a preliminary emitter of length $\ll \lambda$ and emitting the same wavelength as the one to be built later, erected in the same place. It will be shown how the dielectric constant and the conductance of the earth may be derived from six measurements of the electromagnetic field of the preliminary emitter.

The knowledge of these constants enables us to guarantee a definite fieldstrength at the same point of observation when later the definite emitter has been built and when e.g. a total energy of 1 kW has been put into it.

The radiation from the mathematical dipole, mentioned above which has e.g. a moment $= 1/2$, can be measured in several ways, for instance by means of its vertical electric field

$$E_z = |E_z| e^{-j\omega t}$$

or by means of its horizontal magnetic field

$$H_\varphi = |H_\varphi| e^{-j\omega t}$$

($j = \sqrt{-1}$, $\omega = 2\pi\nu =$ circular frequency).

For points above and on the surface of a perfectly conducting earth it would seem (on account of the reflection against the surface of the earth)

as if a dipole of moment 1 was emitting in free space, the dipole of moment 1/2 being assumed close to the earth.

At points on the ground at distances $r \gg \lambda$ from the emitter, $|E_z|$ and $|H_\varphi|$ are then both proportional to $1/r$, so that the products $r |E_z|$ and $r |H_\varphi|$ are constant. In case of a finitely conducting earth those products become functions of r . In order to obtain products which are dimensionless and functions of r/λ we assume that $|E_z|$ and $|H_\varphi|$ are expressed in such units that, multiplied with r/λ , they give products which are constant and equal to unity as soon as we take $\sigma = \infty$.

We need not investigate these units further since we shall never have to use these products themselves but always quotients of two of them.

The above defined products let us call:

$$|E_z| r/\lambda = P_e(r/\lambda),$$

$$|H_\varphi| r/\lambda = P_m(r/\lambda).$$

The radiation from the mathematical dipole on the earth, received in different points of the surface, can be described by means of a Hertzian vectorfunction Π for which Sommerfeld in 1926 derived the formula:

$$\Pi(r) = \frac{e^{jk_1 r}}{r} \left[1 + j\sqrt{\pi} \varrho e^{-\varrho} - 2\sqrt{\varrho} e^{-\varrho} \int_0^{\sqrt{\varrho}} e^{v^2} dv \right] \quad (1)$$

with $k_1 = 2\pi/\lambda$ and in which ϱ is Sommerfelds numerical distance which is generally complex:

$$\varrho = \varrho_0 e^{jb}, \quad (2)$$

$$\varrho_0 = \frac{\pi r}{\varepsilon_2 \lambda} \sin b, \quad (3)$$

$$\tan b = \frac{\varepsilon_2}{2c\sigma\lambda}, \quad 0 < b < \frac{\pi}{2}; \quad (4)$$

$\varepsilon_2 =$ dielectric constant of the earth,

$\sigma =$ conductance (in CGS-units),

$c = 3 \cdot 10^{10} =$ velocity of light.

For the derivation of (1) Sommerfeld was obliged to assume:

$$|k_2^2| = \left| \frac{\varepsilon_2 \omega^2 + j\omega \sigma_2}{c^2} \right| \gg k_1^2 = \frac{\omega^2}{c^2}, \quad (5)$$

$$(\sigma_2 = 4\pi c^2 \sigma).$$

Van der Pol and Niessen derived for the same problem by quite different means (operational methods) a rigorous solution independent of the assumption $|k_2^2| \gg k_1^2$:

$$\Pi(r) = \frac{h^2}{k_1 k_2} \frac{k_2^2}{k_1^2 - k_2^2} \left[\frac{k_2}{k_1} \frac{e^{jk_1 r}}{r} - \frac{k_1}{k_2} \frac{e^{jk_2 r}}{r} + jh \int_{k_1}^{k_2} \frac{e^{irs}}{\sqrt{s^2 - h^2}} ds \right], \quad (6)$$

in which

$$h^2 = \frac{k_1^2 k_2^2}{k_1^2 + k_2^2}.$$

For $|k_2^2|/k_1^2 \gg 1$ they were able to derive Sommerfelds formula (1) from their formula (6).

On account of

$$\frac{k_1^2}{|k_2^2|} = \frac{\sin b}{\varepsilon_2} = \frac{2 q_0}{k_1 r},$$

we see that Sommerfelds formula may be used only in the case when

$$k_1 r \gg 2 q_0.$$

In the case of a short wavelength or insufficient conductance $\sin b$ will approach unity and the restriction (5) will no longer be satisfied. Then correction terms with $k_1^2/|k_2^2|$ or $2 q_0/(k_1 r)$ are to be added to the Sommerfeld formula.

If the uncorrected Sommerfeld formula is written in the form:

$$\Pi = \frac{e^{j k_1 r}}{r} (\Re_S + j \Im_S), \quad (7)$$

\Re_S and \Im_S being real functions of q_0 and b , the corrected formula will be of the form:

$$\Pi = \frac{e^{j k_1 r}}{r} (\Re + j \Im)$$

with

$$\Re = \Re_S + \text{terms with } \frac{2 q_0}{k_1 r}, \quad (8)$$

$$\Im = \Im_S + \text{terms with } \frac{2 q_0}{k_1 r}.$$

The explicit form of these last terms will not concern us in the following.

For the evaluation of P_e and P_m we must first calculate the forces:

$$E_z = - \frac{\partial^2 \Pi}{\partial r^2} - \frac{1}{r} \frac{\partial \Pi}{\partial r}, \quad (9)$$

$$H_\phi = \frac{j c}{\omega} k_1^2 \frac{\partial \Pi}{\partial r}. \quad (10)$$

Where Sommerfelds formula holds we can substitute (7) for Π and in evaluating the forces we may neglect $1/k_1 r$ and $(1/k_1 r)^2$ against 1 if $r \gg \lambda$ (i.e. $k_1 r \gg 1$). We then find P_e and P_m proportional to $\sqrt{\Re_S^2 + \Im_S^2}$.

For $q_0 < 1$ we obtain from Sommerfelds expansion of Π in powers of r the wellknown formulae:

$$\begin{aligned} \Re_S = 1 - 2 q_0 \cos b + \frac{(2 q_0)^2}{1 \cdot 3} \cos 2 b - \frac{(2 q_0)^3}{1 \cdot 3 \cdot 5} \cos 3 b + \dots \\ + \sqrt{\pi q_0} \left(\sin \frac{b}{2} - q_0 \sin \frac{3}{2} b + \frac{q_0^2}{2!} \sin \frac{5}{2} b - \dots \right), \end{aligned} \quad (11)$$

$$\begin{aligned} \Im_S = -2 q_0 \sin b + \frac{(2 q_0)^2}{1 \cdot 3} \sin 2 b - \frac{(2 q_0)^3}{1 \cdot 3 \cdot 5} \sin 3 b + \dots \\ + \sqrt{\pi q_0} \left(\cos \frac{b}{2} - q_0 \cos \frac{3}{2} b + \frac{q_0^2}{2!} \cos \frac{5}{2} b - \dots \right). \end{aligned} \quad (12)$$

Likewise for $q_0 \gg 1$ formulae for \Re_S and \Im_S (expressed in q_0 and b) can easily be derived from Sommerfelds expansion of Π in powers of $1/r$:

$$\begin{aligned} \Re_S = -\frac{1}{2 q_0} \cos b - \frac{1 \cdot 3}{(2 q_0)^2} \cos 2 b - \frac{1 \cdot 3 \cdot 5}{(2 q_0)^3} \cos 3 b - \dots, \\ \Im_S = \frac{1}{2 q_0} \sin b + \frac{1 \cdot 3}{(2 q_0)^2} \sin 2 b + \frac{1 \cdot 3 \cdot 5}{(2 q_0)^3} \sin 3 b + \dots. \end{aligned}$$

The general formulae for \Re_S and \Im_S would be:

$$\begin{aligned} \Re_S = 1 - 2 q_0 \int_0^1 e^{(t^2-1) q_0 \cos b} \cos \{ (t^2-1) q_0 \sin b + b \} dt + \sqrt{\pi q_0} e^{-q_0 \cos b} \sin \{ q_0 \sin b - b/2 \}, \\ \Im_S = -2 q_0 \int_0^1 e^{(t^2-1) q_0 \cos b} \sin \{ (t^2-1) q_0 \sin b + b \} dt + \sqrt{\pi q_0} e^{-q_0 \cos b} \cos \{ q_0 \sin b - b/2 \}. \end{aligned}$$

In the method for determining the earth's constants, which we shall explain in § 2, we shall mostly need the formulae (11) and (12) for \Re_S and \Im_S for $q_0 < 1$.

$$\text{For } \sigma = \infty \text{ we have } b = 0, \quad q_0 = 0, \quad \Re_S = 1, \quad \Im_S = 0,$$

so that, using the normalisation mentioned above, we must write:

$$P_e = P_m = \sqrt{\Re_S^2 + \Im_S^2}, \quad \text{when} \quad \frac{|k_2^2|}{k_1^2} \gg 1 \quad \text{and} \quad r \gg \lambda.$$

In the region $2\lambda < r < 4\lambda$ neglect of $(1/k_1 r)^2$ against 1 will be permitted², but not that of $1/k_1 r$ against 1.

These terms with $1/k_1 r$ have a different influence in the calculation of E_z according to (9) than in the calculation of H_φ according to (10).

Taking into account that instead of \Re_S and \Im_S other functions \Re and \Im of the form (8) have to be used as soon as $|k_2^2|/k_1^2 \sim 1$ or $2\lambda < r < 4\lambda$, we find inclusive of the terms with $1/k_1 r$:

$$P_e = \sqrt{\Re^2 + \Im^2 + \frac{4}{k_1 r} \left(\Re_S q_0 \frac{\partial \Im_S}{\partial q_0} - \Im_S q_0 \frac{\partial \Re_S}{\partial q_0} \right)}, \quad (13)$$

$$P_m = \sqrt{\Re^2 + \Im^2 + \frac{2}{k_1 r} \left(\Re_S q_0 \frac{\partial \Im_S}{\partial q_0} - \Im_S q_0 \frac{\partial \Re_S}{\partial q_0} \right)}. \quad (14)$$

In the correction terms with $1/k_1 r$ we simply replaced the values \Re and \Im by \Re_S and \Im_S since terms with $(1/k_1 r)^2$ had already been neglected formerly.

Introducing

$$S = \sqrt{\Re_S^2 + \Im_S^2} \quad (15)$$

we may write:

$$\Re^2 + \Im^2 = S^2 + \frac{\alpha S}{\pi r/\lambda}, \quad \frac{2}{k_1 r} \left(\Re_S q_0 \frac{\partial \Im_S}{\partial q_0} - \Im_S q_0 \frac{\partial \Re_S}{\partial q_0} \right) = \frac{\beta S}{\pi r/\lambda}.$$

The function α has been derived in a previous paper³ but it will not be of much interest to us now, as will be seen.

The function β is in general:

$$\beta = \frac{q_0}{S} \left(\Re_S \frac{\partial \Im_S}{\partial q_0} - \Im_S \frac{\partial \Re_S}{\partial q_0} \right) \quad (16)$$

and especially for $q_0 < 1$ [using (11) and (12)] we find:

$$\begin{aligned} \beta &= \sum_{n=1}^{\infty} (-1)^n \frac{n \sqrt{\pi} q_0^n}{\Pi(n-1/2) S} (\Re_S \sin nb - \Im_S \cos nb) \\ &+ \sum_{n=0}^{\infty} (-1)^n \frac{(n+1/2) \sqrt{\pi} q_0^{n+1/2}}{\Pi(n) S} (\Re_S \cos(n+1/2)b + \Im_S \sin(n+1/2)b). \end{aligned} \quad (17)$$

Introducing the functions α and β we have:

$$P_e^2 \left(\frac{r}{\lambda} \right) = S^2 + \frac{(\alpha + 2\beta) S}{\pi r/\lambda}, \quad (18)$$

$$P_m^2 \left(\frac{r}{\lambda} \right) = S^2 + \frac{(\alpha + \beta) S}{\pi r/\lambda}. \quad (19)$$

§ 2. How to combine the measurements now so as easily to obtain the constants of the earth?

We suppose that we have measured $|E_z|$ as well as $|H_\varphi|$ and that at three distances r_1, r_2, r_3 from the emitter.

We introduce dimensionless numbers r_1^*, r_2^*, r_3^* by means of:

$$r_i = r_i^* \lambda \quad (i = 1, 2, 3).$$

² In case of an infinitely conducting earth this gives an error of less than $1/2\%$ when $r > 2\lambda$.

³ K. F. Niessen, Ann. Physik **29**, 569 [1937].

The instrument used for measuring $|E_z|$ indicates in r_1, r_2, r_3 the numbers

$$N_1^e, N_2^e, N_3^e.$$

and the instrument used for measuring $|H_\varphi|$ indicates the numbers:

$$N_1^m, N_2^m, N_3^m.$$

$|E_z|$ is proportional to N^e and $|H_\varphi|$ to N^m , but the factors of proportionality are different in both cases, different instruments being used.

Thus we obtain:

$$P_e^2(r_i^*) = K [N_i^e r_i^*]^2,$$

$$P_m^2(r_i^*) = L [N_i^m r_i^*]^2,$$

K and L being again different factors of proportionality.

These factors may be eliminated by building the following quotient:

$$\frac{P_m^2(r_1^*) \cdot P_e^2(r_3^*)}{P_m^2(r_3^*) \cdot P_e^2(r_1^*)} = \left[\frac{N_1^m \cdot N_3^e}{N_3^m \cdot N_1^e} \right]^2.$$

Substituting for $P_e^2(r_i^*)$ and $P_m^2(r_i^*)$ their values (18) and (19) we find, neglecting terms with $(1/r_i^*)^2$ against 1:

$$\frac{P_m^2(r_1^*) \cdot P_e^2(r_3^*)}{P_m^2(r_3^*) \cdot P_e^2(r_1^*)} = 1 - \frac{\beta_1}{\pi r_1^* S_1} + \frac{\beta_3}{\pi r_3^* S_3},$$

in which β_i and S_i are the values of β and S for $r = r_i$.

We draw especial attention to the fact, that *the functions α_1 and α_3 have disappeared entirely*.

In the same way we may combine the measurements made at r_2 and r_3 or those made at r_1 and r_2 .

Now we choose the points of observation so, that:

$$r_1/\lambda = r_1^* \sim 1,$$

$$r_2/\lambda = r_2^* \sim 1,$$

$$r_3/\lambda = r_3^* \gg 1,$$

i. e. we take r_1 and r_2 in the region $2\lambda < r < 4\lambda$ and r_3 at a remote distance from it. We shall only make those combinations where measurements made at r_3 are included, which can only be done in two different ways, viz. combining those at r_1 and r_3 :

$$1 - \frac{\beta_1}{\pi r_1^* S_1} = \left[\frac{N_1^m N_3^e}{N_3^m N_1^e} \right]^2 = Q_{13},$$

and those at r_2 and r_3 :

$$1 - \frac{\beta_2}{\pi r_2^* S_2} = \left[\frac{N_2^m N_3^e}{N_3^m N_2^e} \right]^2 = Q_{23}.$$

In the left hand members the term with $\beta_3/\pi r_3^* S_3$ could be omitted owing to $r_3 \gg 1$. The right hand members are dimensionless quantities Q_{13} and Q_{23} produced by the experiments and which will not differ very much from unity. *The difference of Q_{13} and Q_{23} from unity, viz.*

$$1 - Q_{13} \quad \text{and} \quad 1 - Q_{23},$$

are the quantities which will enable us to calculate the constants of the soil in an easy manner. We have

$$\frac{\beta_1}{\pi S_1} = r_1^* (1 - Q_{13}) = V_1,$$

$$\frac{\beta_2}{\pi S_2} = r_2^* (1 - Q_{23}) = V_2,$$

V_1 and V_2 being known numerical values.

Let us suppose that we calculate the quotient $\beta/\pi S$ as a function of ϱ_0 and with b as a parameter, using (16) and (11) and (12) or the corresponding formulae for $\varrho_0 > 1$. We may then draw a curve, whose ordinate is $\beta/\pi S$ and whose abscissa is ϱ_0 , taking a definite value for b as parameter. Repeating this for other values of b we obtain a system of curves.

Only one of these curves will belong to that value of the parameter b which is typical for the soil, above which the measurements at r_1, r_2 and r_3 were made.

The known values V_1 and V_2 must help us to pick out the right curve from the whole system and thus to find the quantity b which is one of the keys for finding ϵ and σ .

If we knew the right curve and saw where its ordinate reached the values V_1 and V_2 we should find that this occurs at the right values ϱ_{01} and ϱ_{02} of the abscissa for which we have, according to (3).

$$\varrho_{01} : \varrho_{02} = r_1 : r_2.$$

If we had taken another curve and had seen where its ordinate reached the values V_1 and V_2 the quotient of the corresponding abscissae would not have been the same as that of the distances r_1, r_2 , where the measurements in the region $2\lambda < r < 4\lambda$ were made.

In principle it is thus possible to pick out the right curve by means of the values V_1, V_2 and the

known ratio r_1/r_2 , but this procedure still takes too much time.

It would be much easier, if we had not every time to check the ratio of the corresponding abscissae but, let us say, their difference. For that reason we shall not draw curves for $\beta/\pi S$ as a function of q with b as a parameter but as a function of $\lg q$ with b again as a parameter.

In this new system of curves we have to find out that curve, the ordinates of which reach the values V_1 and V_2 for two values of the abscissae ($\lg q_{01}$ and $\lg q_{02}$), the difference of which is

$$\lg q_{01} - \lg q_{02} = \lg (r_1/r_2),$$

being a known numerical value. This procedure, in which we have to check a difference instead of a ratio, is very easy.

The best thing to do is to draw on a separate sheet of paper, which must be transparent, a horizontal line of known length equal to $\lg (r_1/r_2)$ and to erect at the ends of that line perpendicular to it two ordinates of length V_1 and V_2 . This transparent sheet of paper is to be laid upon the new system of curves so that the horizontal axes fall together and then by shifting the transparent paper to the right or left it must be brought into a position where its two points with ordinates V_1 and V_2 both fall on the same curve of the new system. The value of b , belonging to that curve, is the right value to be used in (4) in order to have a relation between the required values of ϵ and σ .

Not only has the value of b now been found, but at the same time we find the value of $\lg q_{01}$, which is indicated in the new system of curves by the place where the foot of the ordinate of length V_1 came to rest.

From the value of b and q_{01} , found above, and the known value of $r_1^* = r_1/\lambda$ we at once find by means of (3) the required dielectric constant ϵ_2 of the earth:

$$\epsilon_2 = \frac{\pi r_1^* \sin b}{q_{01}}.$$

From ϵ_2 and b and the known wavelength λ we find by means of (4) the second required constant, the conductance

$$\sigma = \frac{\epsilon_2}{2c\lambda \tan b}.$$

§ 3. Comparison of the new method with others

In this new method we used electric as well as magnetic measurements which were partly made

in the vicinity of the emitter, and all we needed was one system of curves which could be drawn by choosing several values for a parameter b , independent of the distance between observer and emitter. If we should wish to use measurements at other distances from the emitter we should only have to take another transparent sheet of paper. The whole new system of curves (let us call it the logarithmic system) can then again be used quite unmodified. That this greatly simplifies the solution will be clear as soon as we try to find the constants of the earth from electric or magnetic measurements only.

Suppose we never made measurements in the vicinity of the emitter, so that

$$r_1 \gg \lambda, \quad r_2 \gg \lambda, \quad r_3 \gg \lambda.$$

In this case there is no difference between the results of the electric and the magnetic measurements. In the electric case we can thus combine to obtain:

$$\left[\frac{N_1^e r_1^*}{N_3^e r_3^*} \right]^2 = \frac{S_1^2}{S_3^2} \quad \text{and} \quad \left[\frac{N_2^e r_2^*}{N_3^e r_3^*} \right]^2 = \frac{S_2^2}{S_3^2}.$$

The lefthand members are dimensionless values, given by the experiment. The righthand members are functions of b , q_{01} , q_{03} and of b , q_{02} , q_{03} respectively as is seen from (15), (11) and (12) or the corresponding formulae for $q_0 > 1$.

Although we do not know q_{01} , q_{02} and q_{03} [since their formulae contain, according to (3), the unknown constants of the earth] we do know their ratios:

$$\frac{q_{01}}{q_{03}} = \frac{r_1}{r_3}, \quad \frac{q_{02}}{q_{03}} = \frac{r_2}{r_3},$$

for we know at what distances from the emitter the measurements were made.

So we may consider S_1^2/S_3^2 as a function of b and q_{01} , and S_2^2/S_3^2 as another function of the same variables. We must draw a system of curves representing S_1^2/S_3^2 and another system of curves for S_2^2/S_3^2 as functions of q_{01} with b as a parameter. So we already need two systems instead of one as in the method of § 2. This complication is of course not so serious that it cannot be overcome! But a serious drawback is the fact, that for the construction of those systems the ratios r_1/r_3 and r_2/r_3 respectively are to be used implicitly, so that these systems are useless as soon as, in another case, other distances from the

emitter have to be chosen, the ratios of which may have other values.

If we need not pay attention to this drawback, for instance if we only have to determine ϵ and σ once, these two systems would have to be drawn on the same sheet of paper with coincidence of axes for abscissa and ordinate. A thin bar of

known length $\left[\frac{N_1^e r_1^*}{N_3^e r_3^*} \right]^2$ and a second one of

known length $\left[\frac{N_2^e r_2^*}{N_3^e r_3^*} \right]^2$ may then be laid upon

each other parallel to the axis of ordinates in the systems and with their common foot on the axis of the abscissae. This double bar must then be shifted parallel to itself to the left or right into a position where the upper end of the first bar falls on a curve of the system for S_1^2/S_3^2 and the upper end of the second bar on a curve of the system for S_2^2/S_3^2 which curves have to bear the same value of b , otherwise the right position will not yet have been reached. The value of b , found by this procedure, can be used for the calculation of ϵ and σ . The great inconvenience of this method is, as has been said, the necessity of drawing two new systems of curves as soon as measurements are made at other distances from the emitter with other ratios r_1/r_3 and r_2/r_3 .

This drawback still remains when one of the measurements is made in the region $2\lambda < r < 4\lambda$ and only electric or only magnetic fieldstrengths are measured.

In the case

$$r_1 \sim \lambda, \quad r_2 \gg \lambda, \quad r_3 \gg \lambda,$$

for instance, we know the numerical values of

$$\frac{S_1^2}{S_3^2} \left[1 + \frac{a_1 + 2\beta_1}{\pi r_1^* S_1} \right] \quad \text{and} \quad \frac{S_2^2}{S_3^2},$$

being functions of b , q_{01} , q_{03} and of b , q_{02} , q_{03} respectively so that again the special ratios r_1/r_3 and r_2/r_3 are to be included in the construction of the curves.

In order that only one system of curves be required it is necessary to make both electric and magnetic measurements.

Then a factor as for instance S_1^2/S_3^2 appearing in a quotient of electric measurements can be eliminated by dividing it by a quotient of magnetic

measurements in which the factor S_1^2/S_3^2 also appears. But if both $r_1 \gg \lambda$ and $r_3 \gg \lambda$, this would lead to the identity $1 = 1$. Thus we were obliged to take one of the points of observation (not both) in the region $2\lambda < r < 4\lambda$, viz.

$$r_1 \sim \lambda, \quad r_3 \gg \lambda.$$

Then we get an extra factor

$$1 + \frac{a_1 + 2\beta_1}{\pi r_1^* S_1}$$

in the quotient of electric measurements and an extra factor

$$1 + \frac{a_1 + \beta_1}{\pi r_1^* S_1}$$

in the quotient of magnetic measurements and then by dividing the second quotient by the first the result will be:

$$1 - \frac{\beta_1}{\pi r_1^* S_1},$$

containing only one index (here 1) so that for its representation by means of curves no quotients such as r_1/r_3 have to be taken into account.

If both r_1 and r_3 were taken in the neighbourhood of the emitter the result would contain $\frac{\beta_1}{\pi r_1^* S_1}$ as well as $\frac{\beta_3}{\pi r_3^* S_3}$ and the curves would again be influenced by the value of r_1/r_3 . However the value of $1 - \frac{\beta_1}{\pi r_1^* S_1}$ alone cannot give us the two unknown quantities ϵ_2 and σ_2 .

Therefore everything would have to be repeated with measurements at $r_2 \sim \lambda$ and $r_3 \gg \lambda$.

By this manner of reasoning we came to the method explained in § 2 of this paper. Owing to the fact that the same system of curves can always be used, this method lends itself very well to a determination of the soils constants for propagation in different directions from the emitter. In case of a homogeneous earth the measurements in each direction would give the same result for ϵ_2 and σ , but in reality the results will differ to some extent, giving us an insight into the properties of the ground around the emitter, which will be very useful for calculating the fieldstrength to be expected in several points of observation, when afterwards the final antenna will have been built on the same place where the preliminary one had been erected.